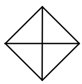


Why

We want to discuss when two sets are the same, and to do so we want to say when all the elements of one set are in another set.

Definition

Denote a set by A and a set by B. If every element of the set denoted by A is an element of the set denoted by B, then we say that the set denoted by A is a *subset* of the set denoted by B.

We say that the set denoted by A is *included* in the set denoted by B. We say that the set denoted by B is a *superset* of the set denoted by A or that the set denoted by B *includes* the set denoted by A.

Every set is included in and includes itself. If the set denoted by B is a subset of the set denoted by A, but B is not A, we call B a *proper subset* of A.

Notation

Let A denote a set and B denote a set. We denote that the set A is included in the set B by $A \subset B$. In other words, $A \subset B$ means $(\forall x)((x \in A) \longrightarrow (x \in B))$. We read the notation $A \subset B$ aloud as "A is included in B" or "A subset B". Or we write $B \supset A$, and read it aloud "B includes A" or "B superset A". $B \supset A$ also means $(\forall x)((x \in A) \longrightarrow (x \in B))$.

Some authors use the notation $\subseteq$ for $\subset$, and use $B \subsetneq A$ to indicate that the set denoted by B is a *proper* subset of the set denoted by A.

Properties

There are some properties that our intuition suggests inclusion should have. First, every set should include itself. We describe this fact by saying that inclusion is *reflexive*.

Proposition 1 (Reflexive)**.** *Every set is included in itself.*

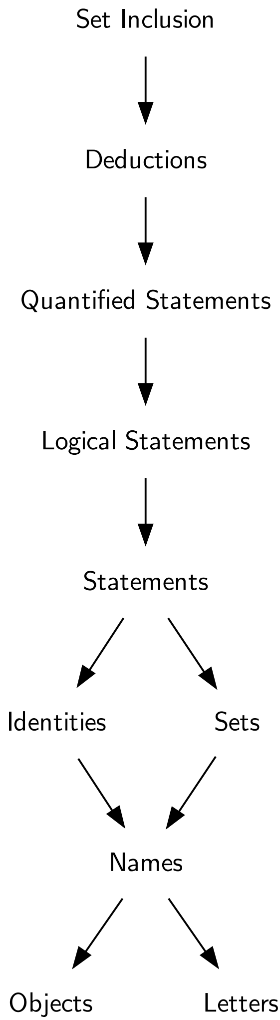
Proof. Suppose A is a set. Then we have $(\forall x)(x \in A \longrightarrow x \in A)$ In other words, $A \subset A$. □

Next, we expect that if one set is included in another, This fact is described by saying that inclusion is *transitive*

Proposition 2 (Transitive)**.** *If a set is included in another, and the latter in yet another, then the first is included in the last.*

Proof. Suppose A, B, C are sets. If $A \subset B$ and $B \subset C$ Thus $A \subset C$ by modus ponens. □

Equality $(=)$ shares these two properties. Let A denote an object. Then $A = A$. Let B and C also denote objects. If $A = B$ and $B = C$, then $A = C$. Of course, inclusion is not symmetric.. Belonging $(\in)$ may be, but need not be reflexive and transitive.


Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters