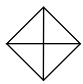


Why

We want to speak of infinite processes, and to do so we define sequences indexed by $\mathbf{N}$. In other words, important families are those indexed by the natural numbers.

Definition

A *sequence* (or *infinite sequence*) is a family whose index set is $\mathbf{N}$ (the set of natural numbers without zero). The *nth term* or *coordinate* of a sequence is the result of the nth natural number, $n \in \mathbf{N}$.[1]

Notation

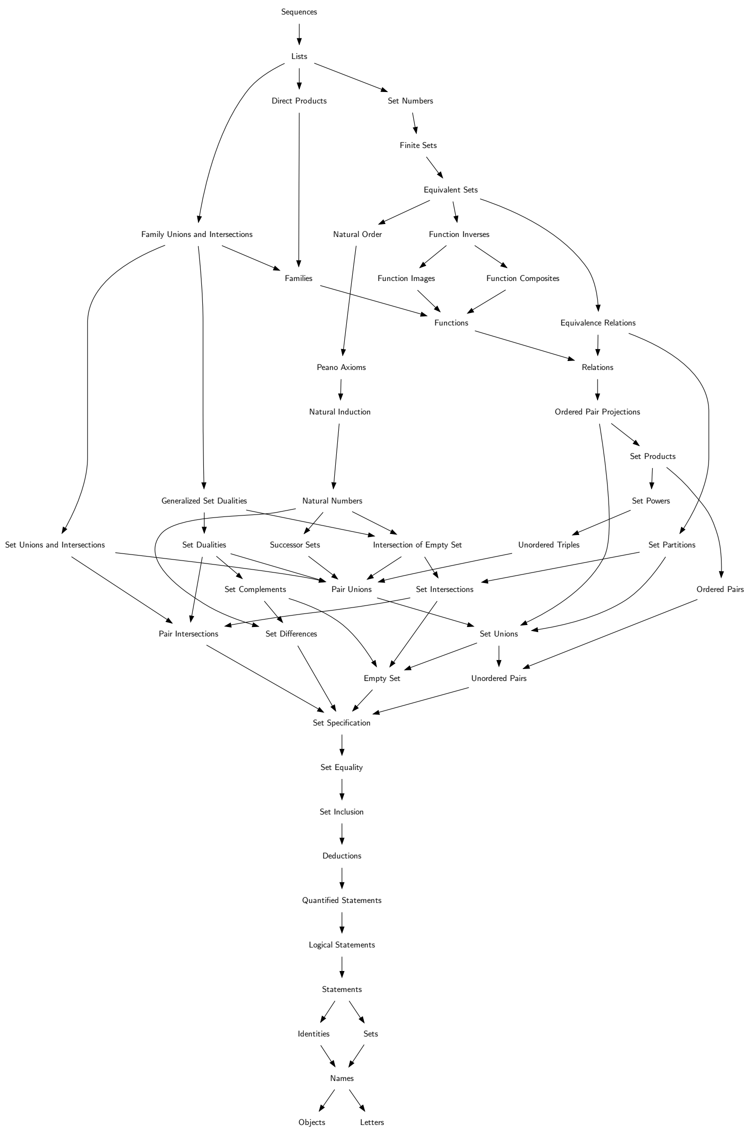
Let A be a non-empty set and $a : \mathbf{N} \to A$. Then a is a (infinite) sequence in A. $a(n)$ is the nth term. We also denote a by $(a_n)_n$ and $a(n)$ by a_n. If $\{A_n\}_{n \in \mathbf{N}}$ is an infinite sequence of sets, then we denote the direct product of the sequence by $\prod_{i=1}^{\infty} A_i$.

Sometimes the set of infinite sequences in A are denoted $A^{\mathbf{N}}$ or A^{∞}.

Natural unions and intersections

We denote the family of the infinite sequence of sets $(A_n)_n$ by $\cup_{i=1}^{\infty} A_i$. Similarly, we denote the intersection of an infinite sequence of sets by $\cap_{i=1}^{\infty} A_i$, respectively.

[1] Future editions may also comment that we are introducing language for the steps of an infinite process.


Sequences
Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Images
Function Composites
Functions
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters