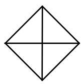


Definition

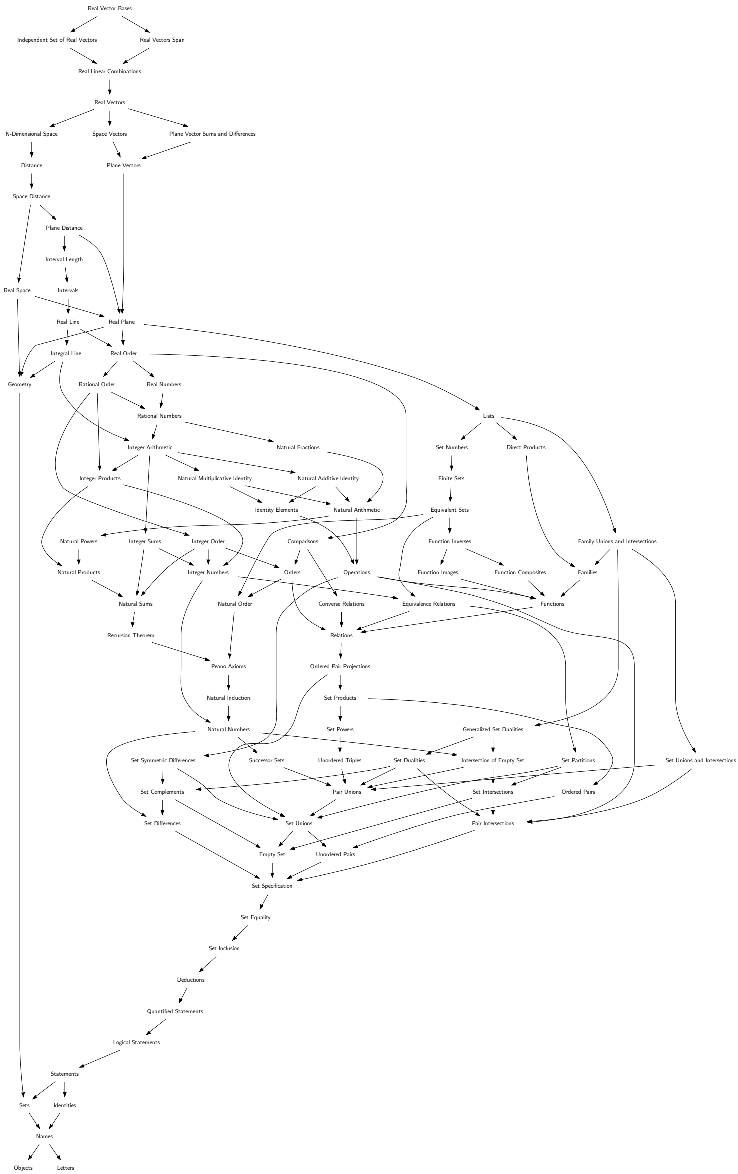
A set of vectors $\{v_1, \dots, v_k\}$ is a *basis* for a subspace $S \subset \mathbf{R}^n$ if

$$S = \operatorname{span}\{v_1, \dots, v_k\} \quad \text{and} \quad \{v_1, \dots, v_k\} \text{ is independent.}$$

This definition captures two competing properties. The first is that the set is large, in the sense that any vector in S can be represented as a linear combination of vectors in $\{v_1, \dots, v_k\}$. Simultaneously, the set is small, in the sense that no vector in the set is a linear combination of the others. In other words, there is no extra vector in the set.

Linear independence is equivalent to uniqueness of representation of the vectors representable as a linear combination of $v_1, \dots, v_k$. In other words, $\{v_1, \dots, v_k\}$ is a basis for S if each vector $x \in S$ can be uniquely expressed as

$$x = \alpha_1 v_1 + \cdots + \alpha_k v_k.$$


Real Vector Bases
Independent Set of Real Vectors
Real Vectors Span
Real Linear Combinations
Real Vectors
N-Dimensional Space
Space Vectors
Plane Vector Sums and Differences
Distance
Plane Vectors
Space Distance
Plane Distance
Interval Length
Real Space
Intervals
Real Line
Real Plane
Integral Line
Real Order
Geometry
Rational Order
Real Numbers
Rational Numbers
Lists
Integer Arithmetic
Natural Fractions
Set Numbers
Direct Products
Integer Products
Natural Multiplicative Identity
Natural Additive Identity
Finite Sets
Identity Elements
Natural Arithmetic
Equivalent Sets
Natural Powers
Integer Sums
Integer Order
Comparisons
Function Inverses
Family Unions and Intersections
Natural Products
Integer Numbers
Orders
Operations
Function Images
Function Composites
Families
Natural Sums
Natural Order
Converse Relations
Equivalence Relations
Functions
Recursion Theorem
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Generalized Set Dualities
Set Symmetric Differences
Successor Sets
Unordered Triples
Set Dualities
Intersection of Empty Set
Set Partitions
Set Unions and Intersections
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Sets
Identities
Names
Objects
Letters