



**Why**

What are some simple functions? Here's one answer: those that only involve addition and multiplication.<sup>1</sup>

**Definition**

A *real polynomial* (or *polynomial*) of degree  $d$  is a function  $p : \mathbf{R} \rightarrow \mathbf{R}$  for which there exists a finite sequence  $a = (a_0, a_1, \dots, a_d)$  so that

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^d.$$

In particular,  $q(x) = ax + b$  for  $a, b \in \mathbf{R}$  is a polynomial of the first degree and  $r(x) = ax^2 + bx + c$  for  $a, b, c \in \mathbf{R}$  is a polynomial of the second degree.

In a sense, these are “simple” functions. We require addition (and subtraction) and multiplication; but no division.

**Properties**

**Proposition 1.** *Let  $p : \mathbf{R} \rightarrow \mathbf{R}$  be a polynomial of degree  $d$ . Then  $p$  is continuous.*

**Proposition 2.** *Let  $p : \mathbf{R} \rightarrow \mathbf{R}$  be a polynomial of degree  $d$ . Then  $p$  has derivatives of all orders. Every derivative of  $p$  is a polynomial.*

**Proposition 3.** *Let  $p : \mathbf{R} \rightarrow \mathbf{R}$  be a polynomial of degree  $d$ . The every derivative of order  $d + 1$  or greater is the constant 0 function.*

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<sup>1</sup>Future editions will modify and expand.



