

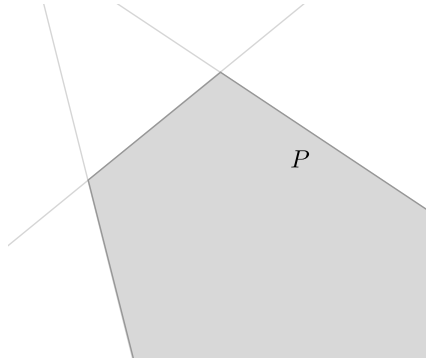


Definition

A *polyhedron* (or *real polyhedron*, or *convex polyhedron*) is a set $P \subset \mathbf{R}^n$ for which there exists $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$ satisfying

$$P = \{x \in \mathbf{R}^n \mid Ax \leq b\}.$$

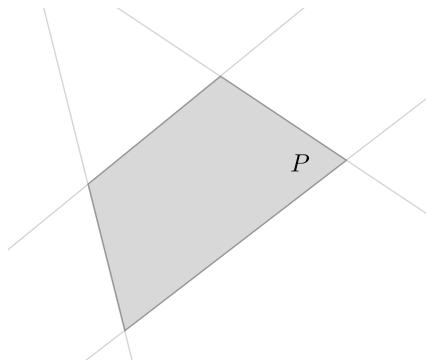
In other words, a polyhedron is an intersection of finitely many halfspaces. If A and b are rational, then P is called a *rational polyhedron*.



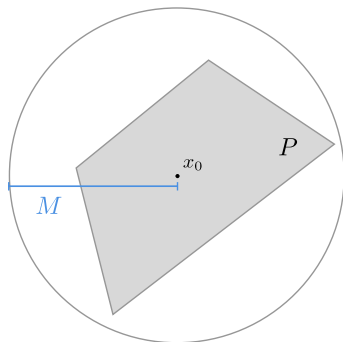
A polyhedron P is a *polytope* (a *real polytope*) if it is *bounded*. In other words, there exists $x_0 \in P$ and $M > 0$ such that

$$P \subset B_M(x_0) = \{x \mid \|x - x_0\| < M\}$$

Here $B_M(x_0)$ denotes the open ball of radius M , as usual.



As usual, the dimension of a polyhedron P is the dimension of the affine hull of P , which we denote by $\dim P$. P is called *full-dimensional* if $\dim P = n$. An equivalent condition for P to be full-dimensional is that there exist an interior point of P (as a subset of \mathbf{R}^n)



Terminology

Caution: some authors have a more relaxed notion of polyhedra, which does not require the polyhedra be convex.

