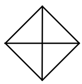


Why

We discuss inferring (or learning) functions from examples.

Definitions

Suppose $\mathcal{U}$ and $\mathcal{V}$ are two sets. A *predictor* from $\mathcal{U}$ to $\mathcal{V}$ is a function $f : \mathcal{U} \to \mathcal{V}$. We call $\mathcal{U}$ the *inputs*, $\mathcal{V}$ the *outputs*, and $f(u)$ the *prediction* of f on $u \in \mathcal{U}$.

An *inductor* is a function from datasets in $\mathcal{U} \times \mathcal{V}$ to predictors from $\mathcal{U}$ to $\mathcal{V}$. A *learner* (or *learning algorithm*) is a family of inductors whose index set is **N**, and whose *nth* term is is an inductor for a dataset of size n.

Notation

Let D be a dataset of size n in $\mathcal{U} \times \mathcal{V}$. Let $g : \mathcal{U} \to \mathcal{V}$, a predictor, which makes prediction $g(u)$ on input $u \in \mathcal{U}$. Let $G_n : (\mathcal{U} \times \mathcal{V})^n \to (\mathcal{U} \times \mathcal{V})$ be an inductor, so that $G_n(D)$ is the predictor which the inductor associates with dataset D. Then $\{G_n : (\mathcal{U} \times \mathcal{V})^n \to \mathcal{V}^{\mathcal{U}}\}_{n \in \mathbf{N}}$ is a learner.

Relations

Functionsare relations, so we might ask if *inferring* relations may be a more general and difficult problem than inferring functions. The following consideration shows that this is *not* the case.

A *relation inductor* is a function from finite datasets in $\mathcal{U} \times \mathcal{V}$ to *relations* on $\mathcal{U} \times \mathcal{V}$. Suppose R is a relation between $\mathcal{U}$ and $\mathcal{V}$. Suppose the function $f : \mathcal{U} \times \mathcal{V} \to \{0, 1\}$ is such that

$$f(u, v) = 1 \quad \text{if and only if} \quad (u, v) \in R$$

Given f we can find R, and given R we can find f. Thus, instead of learning the *relation* R we can think of learning the *function* f. In other words, if we have an inductor for f, we have a *relation* inductor for R.

Consistent and complete datasets

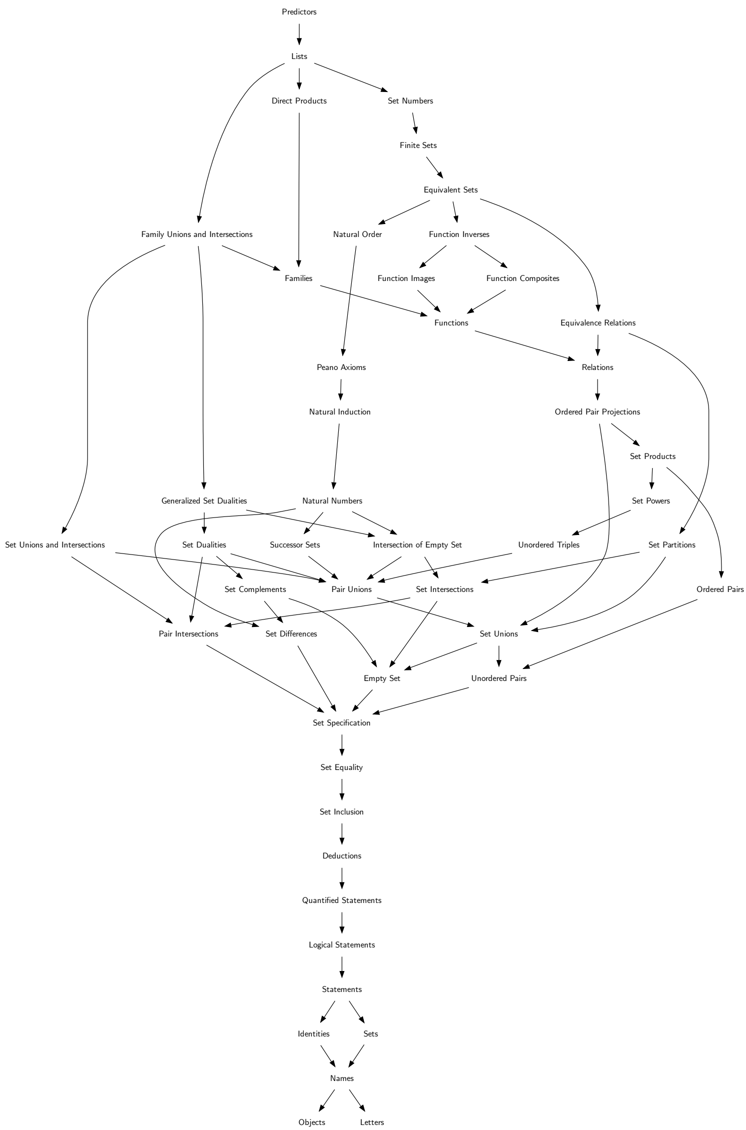
What can a dataset tell us?

Suppose $D = ((u_i, v_i))_{i=1}^n$ be a dataset and $R \subset X \times Y$ a relation. D is *consistent with* R if $(u_i, v_i) \in R$ for all $i = 1, \dots, n$. D is *consistent* if there exists a relation with which it is consistent. A dataset is always consistent (take $R = \mathcal{U} \times \mathcal{V}$). D is *functionally consistent* if it is consistent with a function; in this case, $x_i = x_j \Rightarrow y_i = y_j$. D is *functionally complete* if $\cup_i \{x_i\} = X$. In this case, the dataset includes every element of the relation.

Other terminology

Other terms for the inputs include *independent variables*, *explanatory variables*, *precepts*, *covariates*, *patterns*, *instances*, or *observations*. Other terms for the outputs include *dependent variables*, *explained variables*, *postcepts*, *targets*, *outcomes*, *labels* or *observational outcomes*. An input-output pair is sometimes called a *record pair*.

Other terms for a learner include *supervised learning algorithm*. Other terms for a predictor include *input-output mapping*, *prediction rule*, *hypothesis*, *concept*, and *classifier*.


Predictors
Lists
Direct Products
Set Numbers
Finite Sets
Equivalent Sets
Family Unions and Intersections
Natural Order
Function Inverses
Families
Function Images
Function Composites
Functions
Equivalence Relations
Peano Axioms
Relations
Natural Induction
Ordered Pair Projections
Set Products
Generalized Set Dualities
Natural Numbers
Set Powers
Set Unions and Intersections
Set Dualities
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Partitions
Set Complements
Pair Unions
Set Intersections
Ordered Pairs
Pair Intersections
Set Differences
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters