Why

We want symbols for the words "and", "or", "not", and "implies".[1]

Overview

We call $=$ and $\in$ *relational symbols*. They say how the objects denoted by placeholder names relate to each other in the sense of being and belonging, respectively.

We call $_ = _$ and $_ \in _$ *simple statements*. They denote the simple sentences "the object denoted by $_$ is the object denoted by $_$" and "the object denoted by $_$ belongs to the set denoted by $_$", respectively.

In contrast, the symbols introduced in this sheet are *logical symbols* and we call statements using them *logical statements*.

Conjunction

Consider the symbol $\wedge$. We will agree that it means "and". If we want to say that the two simple statements $a = b$ and $a \in A$ hold simultaneously then we write

$$(a = b) \wedge (a \in A)$$

The symbol $\wedge$ is symmetric, reflecting the fact that a statement like $(a \in A) \wedge (a = b)$ means the same as $(a = b) \wedge (a \in A)$.

Disjunction

Consider the symbol $\vee$. We will agree that it means "or" in the sense of either one, the other, or both. If we want to say that at least one of the simple statements like $a = b$ and $a \in A$ holds, we write

$$(a = b) \vee (a \in A)$$

[1] This sheet does not explain logic. In the next edition there will be several more sheets serving this function.

The symbol $\vee$ is also symmetric, reflecting the fact that a statement like $(a \in A) \vee (a = b)$ means the same as $(a = b) \vee (a \in A)$.

Negation

Consider the symbol $\neg$. We will agree that it means "not". We will use it to say that one object "is not" another object and one object "does not belong to" another object. If we want to say the opposite of a simple statement like $a = b$ we will write $\neg(a = b)$. We read it aloud as "not a is b" or (the more desirable) "a is not b". Similarly, $\neg(a \in A)$ we read as "not, the object denoted by a belongs to the set denoted by A". Again, the more desirable pronunciation goes "the object denoted by a does not belong to the set A." For these reasons, we introduce two new symbols $\neq$ and $\notin$. $a \neq b$ means $\neg(a = b)$ and $a \notin A$ means $\neg(a \in A)$.

Implication

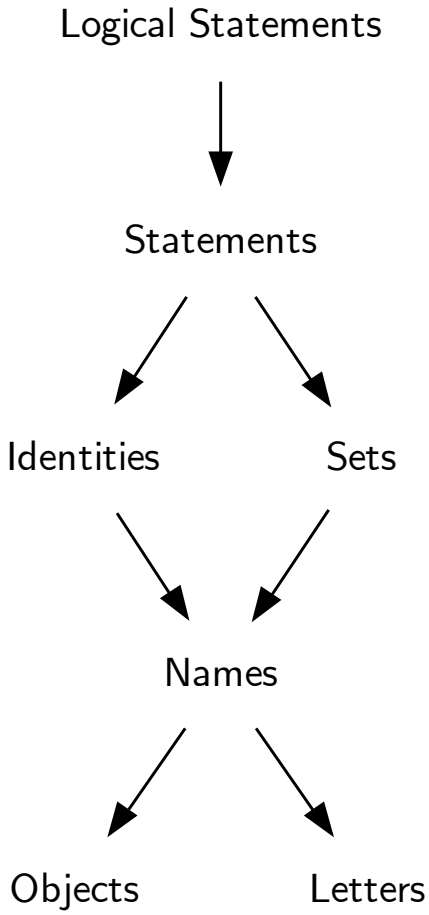
Consider the symbol $\longrightarrow$. We will agree that it means "implies". For example $(a \in A) \longrightarrow (a \in B)$ means "the object denoted by a belongs to the object denoted by A implies the object denoted by a belongs to the set denoted by B". It is the same as $(\neg(a \in A)) \vee (a \in B)$. In other words, if $a \in A$, then always $a \in B$. The symbol $\longrightarrow$ is not symmetric, since implication is not symmetric.

Equivalence

Consider the symbol $\longleftrightarrow$. We will use it to denote the *equivalence* (or *bidirectional implication*) of two logical statments. If we want to say that the logical statements $(a \in A) \Rightarrow (b \in B)$ and $(b \in B) \Rightarrow (a \in A)$ hold simultaneously then we will write

$$(a \in A) \longleftrightarrow (b \in B)$$

We read this symbol aloud as "if and only if", which is sometimes abbreviated with the neologism iff.


Logical Statements
Statements
Identities
Sets
Names
Objects
Letters