Why

We want to discuss making a decision.[1] To discuss decisions, we first must speak of the choices to be made.

Definition

We have a set which includes all possible choices. The set is called the *actions* (or *acts*, *decisions*, *choices* or *designs*).

For each decision, we have a set which includes all possible outcomes. Often the outcomes are uncertain, and associated with the future (see Uncertain Outcomes).

Since we select an action prior to observing the outcome, we want to talk about which actions and outcomes are preferable to others. We call an action-outcome pair a *history*. We refer to the set of action-outcome pairs the *histories*. A *preference* is an order on histories.

A *simple decision problem* is a triple $(A, \{O_a\}_{a\in A}, \preceq)$ in which A is a set of actions, $\{O_a\}$ is a family of sets of outcomes, and $\preceq$ is a preference on the histories $\{(a, o) \mid a \in A, o \in O_a\}$.

Example: party

Consider deciding whether to host a party indoors or outdoors. We are unsure of the weather. We have a set of two actions $A = \text{In}, \text{Out}$ and a set of two outcomes (the same for each action) $O_a = W = \text{Rain}, \text{Shine}$ for $a \in A$. Although many orders on $A \times W$ exist, one such order is

$$(\text{Out}, \text{Shine}) \prec (\text{In}, \text{Rain}) \prec (\text{In}, \text{Shine}) \prec (\text{Out}, \text{Rain}).$$

In other words, having the party outside in the sun is preferred to having it inside when it is raining, but both of these are preferred to having it inside when the sun is shining, and all of these are preferred to having it outdoors in the rain.

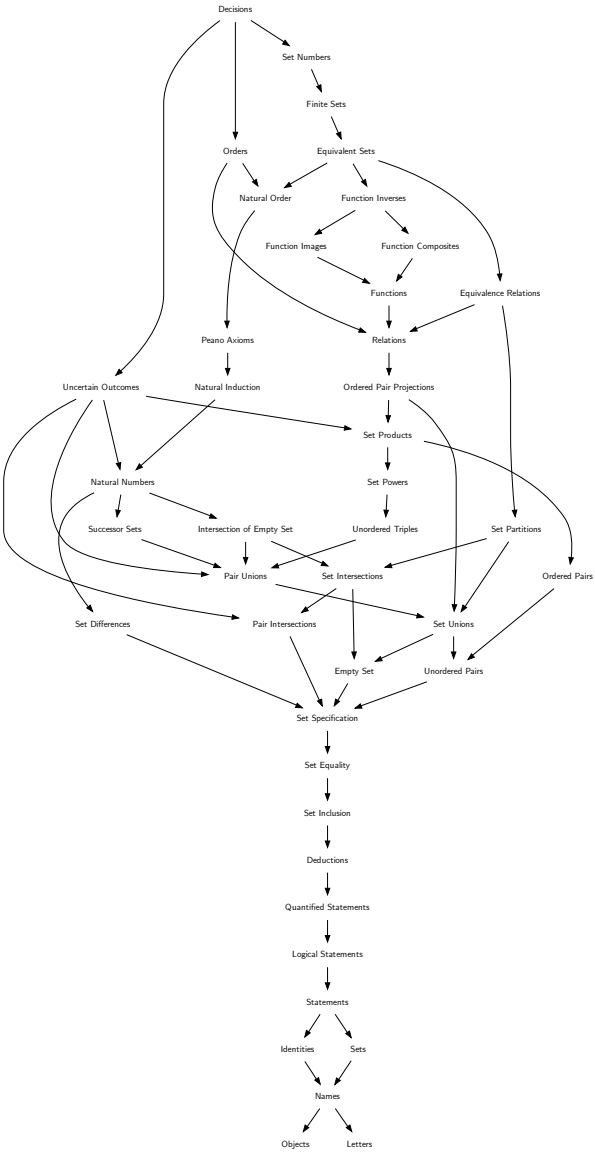
[1] Future editions will expand.

Best actions

Let $(A, \{O_a\}, \preceq)$ be a simple decision problem in which $O_a = S$ for each $a \in A$. Let $o \in S$. An action $a \in A$ is *best for outcome* o if $(a, o) \preceq (a', o)$ for all $a' \in A$. An action $a \in A$ is *best for all outcomes* (or *uniformly best*) if, for all $o \in O$, $(a, o) \preceq (a', o')$ for all $a' \in A, o' \in O$.[2]

In the party example, the best action for RAIN is IN and the best action for SHINE is OUT. On the other hand, there is no uniformly best action. For the action OUT, there is an outcome RAIN, and action-outcome pair (IN, RAIN) so that (IN, RAIN) $\prec$ (OUT, RAIN). In other words, OUT is not uniformly best, because there is an outcome, RAIN for which the action IN is preferred. Similarly for IN.

[2]Future editions will clarify, and may remove this section.


Decisions
Set Numbers
Finite Sets
Orders
Equivalent Sets
Natural Order
Function Inverses
Function Images
Function Composites
Functions
Equivalence Relations
Peano Axioms
Relations
Uncertain Outcomes
Natural Induction
Ordered Pair Projections
Set Products
Natural Numbers
Set Powers
Successor Sets
Intersection of Empty Set
Unordered Triples
Set Partitions
Pair Unions
Set Intersections
Ordered Pairs
Set Differences
Pair Intersections
Set Unions
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters