Why

Given multiple orders, can we combine them?

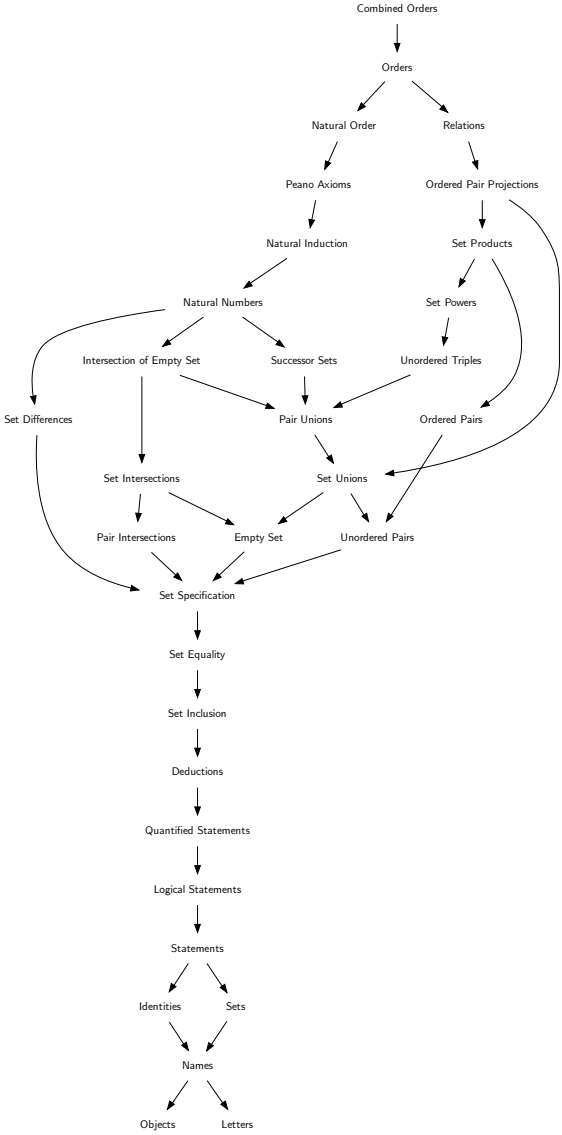
Discussion

Suppose we have two orders $\prec_1$ and $\prec_2$ on A. Define $\prec$ by $a \prec b$ if and only if $a \prec_1 b$ and $b \prec_2 a$. Notice that $\prec$ is reflexive, transitive, and antisymmetric, and so it is a partial order. Call it the *combined order*.

Here's the rub. Even if $\prec_1$ and $\prec_2$ are total, $\prec$ may not be total.[1] Consider the basic case $A = \{a, b\}$ and $\prec_1 = \{(a, a), (a, b), (b, b)\}$ and $\prec_2 = \{(a, a), (b, a), (b, b)\}$ Then $\prec = \{(a, a), (b, b)\}$, a partial order, to be sure, but not really any order at all.

There is not anything to be done about it, it is a fact. Total orders do not (necessarily) induce total combined orders.

[1] Future editions will include and expand.


Combined Orders
Orders
Natural Order
Relations
Peano Axioms
Ordered Pair Projections
Natural Induction
Set Products
Natural Numbers
Set Powers
Intersection of Empty Set
Successor Sets
Unordered Triples
Set Differences
Pair Unions
Ordered Pairs
Set Intersections
Set Unions
Pair Intersections
Empty Set
Unordered Pairs
Set Specification
Set Equality
Set Inclusion
Deductions
Quantified Statements
Logical Statements
Statements
Identities
Sets
Names
Objects
Letters